

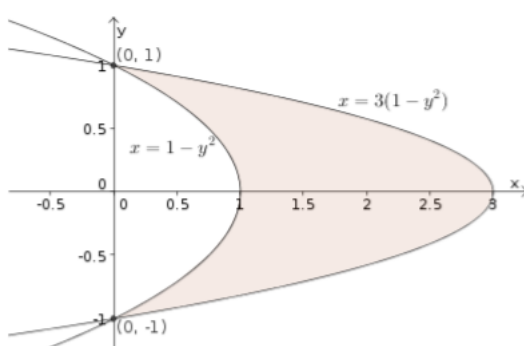
Worksheet for October 17

Problems marked with an asterisk are to be placed in your math diary.

(1.*) Find the volume of the region in \mathbb{R}^3 bounded by the graphs of $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$.

(2.*) Consider the transformation $G(u, v) = (v(1 - u^2), u)$. $\text{Jac}(G) = \det \begin{pmatrix} -2uv & 1 - u^2 \\ 1 & 0 \end{pmatrix} = -(1 - u^2)$.

Show G takes the rectangle $-1 \leq u \leq 1$, $1 \leq v \leq 3$ in the uv -plane to the region below in the xy -plane.



Convince yourself of this by using graphing software (e.g., Desmos) as follows. Graph each of the parabolas $G(u, 1)$, $G(u, 2)$, $G(u, 3)$, $G(u, 4)$, $G(u, 5)$. Conclude that for each fixed v_0 , as u varies from -1 to 1 , $G(u, v_0)$ is that portion of the corresponding parabola lying in the shaded region.

(3.*) Use the change of variables theorem to calculate $\int \int_D \frac{y^2}{x} dA$, for D the shaded region in problem 2.